MATHEMATICS AND BIBLE STUDY

Several times have I made the statement that true Bible study resembles the study of mathematics. It is difficult to show examples since very few people have sufficient background in mathematics. However, here are two mathematical examples that should be within the reach of anyone with a only a knowledge of high school algebra. It is in three parts: Part 1 sets forth the modern axiomatic approach to mathematical analysis. The particular example I have chosen is the set of axioms underlying ordinary arithmetic and elementary algebra. Part 2 gives an algebraic example of a common and very useful function, the *exponential*. Part 3 analyzes a few Biblical ideas to show how some of these principles have their analog in Scriptural analysis.

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PART I: THE AXIOMATIC APPROACH:

Euclid, the famous Greek geometer of the fourth century B.C., collected practically all that was known in his day of plane geometry and systematically presented it in his *Elements*. The one feature that set his work apart from all others was the exploitation of axioms, or postulates, as the foundation on which to construct his system. For example, two points determine a line; a straight line is the shortest distance between two points; two lines intersect in exactly one point; etc. No attempt was made to prove these axioms; they were assumed as intuitively true, having no need of any sort of logical justification. However, theorems had to be proven by extending a logical series of steps from the theorem back to the axioms. If each step were shown to logically follow from the previous one, and if the final step was one or more of the axioms, the proof was considered complete. Of course, what Euclid really did was to first prove very simple, almost obvious, theorems. These, in turn, were used to prove more advanced and less obvious statements, until the whole system was proven, hundreds and thousands of statements. Such as, for a simple example, the sum of the interior angles in a triangle being equal to 180^{o}.[1] A more advanced theorem, and which is certainly not obvious to the average person, is the Pythagorean in which the sum of the squares of the legs of a right triangle is equal to the square of the

[1] In fact, this one is so simple it is often accepted as an axiom, and one of the other axioms is then considered as a theorem to be proven. I hesitate to explore this aspect of the subject, however, because it would take us away from the main thrust of this essay.

hypotenuse. A yet more advanced theorem, and even less obvious, states that any angle (emphasis on any) inscribed in a semi-circle is a right angle. (Carpenters and woodworkers make much use of these last two theorems.)

It was many centuries, however, before the same axiomatic approach was used in any other branch of mathematics. At last, mathematicians ran into several puzzles that forced them to come to grips with the very foundations of their thinking. One result of this was the establishment of axioms for common arithmetic. I will not attempt to set forth all the axioms that were proposed, but will limit myself to a useful subset, and to show a simple example or two of how they are used. In so doing, several ideas will be illustrated that will have a decided bearing on our approach to Bible study.

To give this discussion some direction, I wish to go far enough to prove that the product of two negative numbers is a positive. This has always been troublesome to students when they first encounter it, for two reasons: negatives themselves are artificial constructs, and how to define multiplication of negatives is very abstract and not at all obvious. In fact, I corresponded for a number of years with an Englishman who disagreed with the above statement, maintaining that it made no sense whatsoever, and that the product of two negatives had to be another negative. (He claimed to be an electronics engineer, but I had my doubts.)

DEFINITIONS OF THE ELEMENTS:

Since we are concerned here with common arithmetic and algebra (as usually taught in high school), we must begin by defining the elements of our system.

1. **Number:** many attempts have been made by logicians to define the concept of number. I will assume that this is sufficiently intuitive to most people as to need no further explanation. We note, however, that the concept of unity, or "one," results from our realization that we, as an individual, are distinct from the rest of the world. Our sense of our own individuality gives us the idea of "one." To account for the rest of the population, we gradually come to accept the idea of other numbers. These numbers are called the "natural" numbers, or the "counting" numbers, by mathematicians.

2. **Operations:** we assume we can combine any two numbers to give a third number using any of four basic operations—addition, multiplication, subtraction, and division.

These, then, form the basic elements, or raw materials from which we will construct our system. We now examine the four operations individually.

ADDITION:

Let us begin where most children begin, with addition. Addition is an operation on two numbers, called the addend and the augend. The result is called the sum.

Closure: One axiom we accept without proof is that any two numbers can be added to yield yet a third number. This is called the *property of closure*, and merely guarantees us that we shall create no strange and alien objects by simply adding two numbers. In general, for the given numbers **a** and **b**,

a + b = c, where c is a number.

Commutative Law: We further assume that it makes no difference, for two numbers **a** and **b**, whether we add **a** to **b** or **b** to **a**; the sum is the same. That is, the operation "commutes" back and forth between the two numbers. For example, 3 + 5 = 5 + 3. In general,

a + b = b + a

Associative Law: When adding three numbers, **a, b,** and **c**, it makes no difference which two we add together first before adding the remaining number to the sum. That is, the middle number can associate itself with either neighbor. Suppose we wish to add the numbers 3, 4, and 5. We can first add 5 to 4 obtaining 9, then add this to 3 to get 12. Or, we could add 4 to 3 to get 7, then add 5 to 7 to get 12. Stated in general,

a + (b + c) = (a + b) + c

Additive Identity: We assume the existence of an element, **0**, which, if it is added to any number, results in that same number. That is, it preserves the identity of the number to which it is added. (We commonly say that it adds nothing to the number.) That element is, of course, zero, called the "additive identity."

$$a + 0 = a$$

Commutativity permits us to also state:

$$0 + a = a$$

SUBTRACTION:

Subtraction may be defined in terms of addition as follows:

Given two numbers **a** and **b**, with the first greater than or equal to the second, the difference **a - b** is defined to be x such that

$$x + b = a$$

Thus, 5 - 3 = 2 because 2 + 3 = 5. Subtraction is the inverse of addition: it "undoes" whatever addition does.

However, this definition is true only when the first number is greater than or equal to the second. To make subtraction possible for all numbers regardless of their relative magnitudes (for example, 3 - 5), we must introduce the negative numbers. Negative numbers are usually defined as follows:

The closure property, which simply says that the result of adding any two numbers is another number, has a converse: every number must be the result of adding two numbers.

I like to compare these two ideas to human reproduction. Every human male-female couple (ignoring such trivialities as infertility or immaturity) can produce only another human, not some other creature. This may be compared to the closure property. We also know that every human is the child of a human male-female couple. This is analogous to the converse that every number is itself the sum of two numbers.

For example, 5 can be obtained by adding 1 and 4 (which is only one possibility, but all we wish to do is show that the axiom is true in at least one way). In like fashion, any number greater than 1 can be obtained by adding 1 to an appropriate number. The number 1 itself can be obtained by adding 1 to zero.

But what about zero? How can zero be obtained by adding two numbers? (Other than by adding zero to itself, which is trivial.) Restating this, what two numbers can we add to obtain zero for the result? For the sake of simplicity, let us designate one of these two numbers as 1. We then ask, what must **x** be such that **1 + x = 0?**

Obviously, x must be -1, so that 1 + (-1) = 0. In general, for any number **a,**

a + x = 0 means x = -a.

This negative number is called the additive inverse of **a;** that is, when it is added to its mate yields the additive identity **0.** It is what makes subtraction possible.

Thus, for every natural number, there exists a negative such that when the two are added together, the result is 0. This, in turn, defines subtraction for every possible combination—and permutation—of two numbers, and, at the same time, gives us a whole new set of numbers, the negatives.

The set of numbers so far defined, that is, the natural numbers, their negatives, and zero, is called the "integers."

MULTIPLICATION:

Another operation between numbers can be defined, called multiplication. Let us define some terms. The two numbers to be multiplied are called factors. One of them is called the multiplier and the other the multiplicand. The result is called the product.

Closure: Just as for addition, we state that the result of multiplying two numbers is another number. I can hear some reader now exclaim, "Of course. Why waste our time with that?" How do I know you said that? Because I said exactly the same thing when these ideas were first introduced to me several years ago. I can partly answer your objections by stating that there are mathematical systems in which this axiom is not true. For example, in vector analysis, we define two kinds of multiplication: the scalar product and the vector product. If two vectors are combined into a scalar product, the result is not a vector, but a number. In the vector product, the result is a vector all right, but does not lie in the same plane as the two components. Hence, in either case, we end up with something that is not a member of the set of elements we began with. Therefore, it is not a triviality to say that the product of two numbers is also a number, just as it is not a triviality that the product of human reproduction is another human.

Commutative Law: The commutative law says that either of the two factors may be the multiplier, yielding identical products. Thus, 2 X 3 = 3 X 2. Envision the first as 2 rows of 3 each, and the second as 3 rows of 2 each. In general,

ab = ba

Once again, I must point out that this is not trivial. In matrix algebra, for example, the commutative law is not in general true (though if one suitably restricts the matrices, it can be made so). In other words, given two arbitrary matrices **A** and **B**, **AB** does not normally equal **BA.** One might think that this would cripple the system to such an extent that it would be worthless. Such is far from the case: matrices are some of the most useful and practical mathematical entities ever invented. I simply love matrix algebra.

Associative Law: This states that for any three numbers to be multiplied together, we may proceed by multiplying any two of them together first, then multiplying the result by the third number.

a(bc) = (ab)c

Thus, 3 X (5 X 7) = 3 X (35) = 105; but

(3 X 5) X 7 = (15) X 7 = 105

Multiplicative Identity:

There exists an element such that any number multiplied by it leaves that number unchanged—that is, preserves its identity. That element is, of course, 1, which is called the “multiplicative identity.”

a X 1 = a

and commutativity gives us also

1 X a = a

DIVISION:

The operation of division will be handled in much the same way as subtraction. It is the inverse operation of multiplication, that is, it "undoes" whatever multiplication does.

For two numbers, **a** and **b,** the quotient **q = a/b** is defined such that **bq = a.**

For example, 12/4 = 3 because 4 X 3 = 12. Division is the inverse of multiplication because it "undoes" whatever multiplication does.

But, all of a sudden, we have a problem. Just as in subtraction wherein certain problems could not be solved until we introduced the negatives, so in division certain problems have no solution until we introduce the fractions. For example, 13/4 remains undefined as long as we insist on whole number quotients. Fractions, otherwise known as rationals, are usually constructed as follows:

Every number except zero is assumed to have a multiplicative inverse such that the product of the two numbers is 1. Let the inverse of a be represented as **1/a.** Hence,

a X (1/a) = 1 and commutativity permits also

(1/a) X a = 1

According to the definition of division above, this means that, for **q = 1/a** to exist, we must have **aq = 1.**

Try as we may, however, we can find only one number for which we can obtain its inverse from among the integers: the number 1 itself; the inverse of 1, written as 1/1, is just 1, since 1 X 1 = 1. (It is its own inverse.) But that is not much help. We wish to find, for any **a**, an inverse **x** such that

$$ax = 1$$

It turns out that we must introduce another class of numbers we have not hitherto encountered: the *fractions*, which are more precisely called by mathematicians the *rationals*. Thus, 1/3 or one-third, is a rational number, so called because it is the ratio of two integers. They are just as difficult for students to comprehend as the negative numbers, perhaps more so. Fractions are usually pictured as portions of a whole, such as a slice of pie is said to be perhaps 1/6 of the whole pie. The difficulty is that fractions cannot be used in counting; as Lewis Carroll pointed out when he said, "try looking at the moon one-third of a time." The parallel with subtraction should be obvious: just as negatives are introduced to solve certain problems, acquiring their own characteristics, so are the fractions useful for solving certain problems and will acquire their own individuality.

However, most school children are taught about fractions before negative numbers, and this so early on that it soon becomes second nature to talk about them.[2]

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[2] Although, in my experience, very few high school graduates have much of an understanding of fractions. What they know is mostly by rote, not by logic.

It can be said quite simply: for any number **a** (with one exception to be discussed below), there exists a multiplicative inverse, **1/a**, such that the product of the two is the multiplicative identity, **1.** Saying it symbolically,

a (1/a) = 1 and commutativity also permits

(1/a) a = 1

The exception is zero itself: **1/0** is meaningless. Another way of saying this is that it is illegal to divide by zero. So now we have division defined: if **q = b/a** for non-zero **a**, then **b = aq.**

Before proceeding, let us briefly summarize and compare the two operations addition and multiplication.

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Addition

Closure: **a + b = c**, where **c** is a number

Commutative: **a + b = b + a**

Associative: **a + (b + c) = (a + b) + c**

Inverse, Additive :*

a + (-a) = 0 or

(-a) + a = 0

Exception: there are none.

Subtraction:

a - b = c means **a = b + c**

Multiplication

Closure: **ab = c**, where **c** is a number

Commutative: **ab = ba**

Associative: **a(bc) = (ab)c**

Inverse, Multiplicative *

a (1/a) = 1 or

(1/a) a = 1

Exception: **1/0** undefined

Division:

a/b = q means **a = bq**

* The commutative law permits either expression.

The symmetry between these two groups is remarkable. Notice that one may interchange addition with multiplication and subtraction with division, if one also interchanges zero with one and the additive with the multiplicative inverse. The only non-symmetrical element is the exception that division by zero is not allowed. Mathematicians label these two systems *groups*, an additive group and a multiplicative group. Both are said to be commutative, because there are groups that are not commutative.

THE LAW OF DISTRIBUTION:

And now, for a most remarkable law, the distributive law. This law ties the two groups together so that there is a sort of communication between them. It can be stated as follows:

Let two numbers, **b** and **c**, be added, and the sum further multiplied by **a**. The result is the same as if **b** and c are first individually multiplied by **a,** then the products added together.

Example:

$3\,(2 + 5) = 3\,(7) = 21$, formed by first adding the parenthetical expression..

But we could multiply first:

$3 \times 2 + 3 \times 5 = 6 + 15 = 21.$

In general:

$$a\,(b + c) = ab + ac.$$

That is, **a** distributes its multiplicative powers over the two terms, **b** and **c.** This law makes possible that horrible student exercise despised by most novice algebraists known as factoring. For, suppose we have **ab + ac.** We can factor out a giving **a (b + c).** Of course, the expressions may be quite involved:

$$(x - 1)^2 + (x^2 - 1) + (x^3 - 1)$$

where the common factor **(x - 1)** appears in every term. It may therefore be factored out to give:

$$(x - 1)\ [(x - 1) + (x + 1) + (x^2 + x + 1)]$$

To prove this, simply multiply each parenthetical term within the brackets by **(x - 1)**; the distributive law gives us permission to do this. But we shall not be too much concerned with exercises of that complexity.

Notice one thing: the distribution is that of multiplication over addition. There is another distributive law, that of addition over multiplication, that could be stated as follows:

$$a + (bc) = (a + b)(a + c)$$

This is obtained from the first distributive law by interchanging addition and multiplication.

This law is not true for arithmetic. For example, 2 + (3 X 5) is not equal to (2 + 3) X (2 + 5) because the first expression gives 17 and the second gives 35. But both distributive laws are true for a special algebra known as Boolean Algebra, also called the algebra of logic, which is widely used in computer technology.

I ought to mention here that several axioms have been omitted, such as: **if equals are added to, or multiplied by, or subtracted from, or divided by, equals, the results are equal.** Another very valuable axiom is this:

Equals may be substituted for equals in any expression. (This is one which will prove to be of much use in Bible study.)

There are others having to do with unequal numbers, such as: **if a is less (or greater) than b, and b is less (or greater) than c, then a is less (or greater) than c.** Finally there is one, called the reflexive law of equality, that says **if a = b, then b = a.** I find this one to be particularly useful in Bible Study.

There are still others of like nature, but I am not desirous in stating the whole lot of them; rather, I wish to exhibit how the ones stated here can be used to prove a certain theorem.

We will use these axioms to prove that ***the product of two negatives is a positive.***

THEOREM: The product of two negative numbers is a positive number.

This proof will leave out only one step, but it is so trivial that I doubt if any one would object. I'll point it out when the time comes.

Lemma:

First, we prove a simple lemma:

The product of any number and zero is zero. In other words, we have an interaction between the two groups: we use the operation from the multiplicative group coupled with the identity from the additive group, and desire to see what the result may be. We wish to prove that **a X 0 = 0.**

In order to do this, consider the expression **a + (a X 0).** Replace a in the leftmost term with **a X 1**. We can do this because multiplying by 1 leaves the number unchanged. Our expression then becomes,

a X 1 + a X 0

Use the distributive law to factor out a from both terms. We get,

a (1 + 0)

The quantity in parentheses may be simplified to 1 because adding zero to any number leaves it unchanged. Our expression now is,

a (1) or, omitting the unnecessary parentheses,

a X 1 = a

because multiplying by 1 leaves the number unchanged.

So now we have reduced the original expression to **a:**

a + (a X 0) = a

Hence, the quantity in parentheses, whatever it is, acts exactly like the additive identity zero in that adding it to the first term leaves it unchanged. Hence, **a X 0 = 0.**

Normally, the thorough mathematician would go ahead at this point and prove that there exists only one such element. If there should happen to be two elements with the additive identity property, then our proof would be defective. The proof is quite easily stated but is a little tricky to understand. However, I think it is intuitively clear that there is only one element that has the property of our additive identity. I must point out, though, that there are mathematical systems where there are many zeroes. They look different and are different from one another, but all possess the same property of leaving unchanged whatever they are added to.

Proof Of Theorem:

Our second step in the proof of the theorem consists of evaluating the following expression by two different methods, and, since it can have only one value, the two results, though looking very different, are in fact identical.

Consider the following expression:

$$e = ab + a(-b) + (-a)(-b)$$

First, by utilizing the associative law for addition, group the first two terms together:

$$e = [ab + a(-b)] + (-a)(-b)$$

(which also may be grouped as $e = ab + [a(-b) + (-a)(-b)]$ because of the distributive law.)

Utilizing the distributive law, factor out **a** from the first two terms:

$$e = a [b + (-b)] + (-a)(-b)$$

Now, utilizing the property of the additive inverse, combine the bracketed terms

$b + (-b)$ to get **0** and substitute:

$$e = a [0] + (-a)(-b)$$

But we now know that $a [0] = 0$

Hence, we have

$$e = 0 + (-a)(-b)$$

But adding zero to a quantity leaves it unchanged. Hence, our expression reduces to

$$e = (-a)(-b)$$

Now let us go back to the original expression and regroup it, as in the second expression, by placing the second pair of terms in brackets. We are justified in this by the associative law of addition:

$$e = ab + [a(-b) + (-a)(-b)]$$

The first step is to make use of the commutative law of multiplication to switch the order of the factors in each of the bracketed terms:

$$e = ab + [(-b)a + (-b)(-a)]$$

Now, by using the distributive law, factor out (-b) from each term:

e = ab + (-b)[a + (-a)]

As before, the sum of the two bracketed terms is zero:

e = ab + (-b) X [0]

But by the lemma, the second term is precisely zero, therefore adding it to ab does nothing; so our expression reduces to

e = ab

By our first reduction we obtained **e = (-a)(-b)**. But our second reduction obtained **e = ab**. Since e can have only one value, that is, since **e = e,** then surely **ab = (-a)(-b).** Or, the product of two negatives is the same as the product of two positives, that is, positive.

Example: -3 X -5 = +3 X +5 = +15.

Note: the plus sign is usually omitted for positive numbers.

Comment:

The English gentleman with whom I corresponded kept saying that it was far more natural to define the product of two negatives as a negative. He would have the above result to be **-15**. I explained to him that, indeed, if we had simply wanted to define multiplication without regard to anything else, we could have done so, but we would have contradicted other and far more intuitive assumptions. Having such a choice, we kept the original postulates and so defined multiplication of negatives that it did not contradict them.

It was a wise decision. It has been vindicated infinitely many times in the entire realm of mathematics and the natural sciences. Once again, let me state that if we defined multiplication of negatives some other way, we would in short order find that we had violated some of the most basic and elementary concepts, concepts that have never been doubted nor denied, at least for simple arithmetic. Our definition is completely harmonious with all the rest of arithmetic and preserves other truths of which there is no doubt. That this is basic to Bible study will become evident later on, but let me say this for the moment: if we advance an interpretation of some Biblical passage and find that it violates some principle already established and accepted by leading scholars, we would do well to reexamine our interpretation. If, however, no contradictions can be found and if it supports the bedrock principles already well established, then we may be on to something, even if we can find few scholars that agree to it. Sometimes scholars disagree because of the "not invented here" syndrome, or because of some other bias, but are unable to find any real Scriptural disproof. I have been in contact with both situations and I feel that I have a pretty good understanding of what is involved.

CANCELLATION LAW:

I now introduce one more law, which, though it was not needed in the proof of our theorem, will come in handy at a later point. It is the so-called cancellation law. Quite simply, it says that if two numbers are multiplied by a third, obtaining equal results, the two numbers are themselves equal:

$ab = ac$ means that $b = c$.

We can strike out, or cancel, a from both sides. ***Caution: only if a is not zero!***

We can either choose to prove this from our other axioms, in which case it becomes a theorem (or at least a lemma), or we can simply admit it as an axiom without proof. If we do that, it will be found that some other axiom is unnecessary. I will take the first approach and prove it in terms of other axioms. The proof is quite simple. Subtract **ac** from both sides of the equation:

$$ab - ac = ac - ac = 0$$

Now factor out **a:**

$$a(b - c) = 0$$

Assume **a** is not zero. Then **(b - c)** must be zero, because that is the only way we can get zero when it is multiplied by a non-zero quantity.

$$b - c = 0$$

Now add **c** to both sides:

$$b(- c + c) = 0 + c = c$$

or $$b = c$$

We could have gotten exactly the same result by simply canceling out the common factor **a** from both sides of the original equation. But remember, we can do this only if **a** itself is not zero, for any number multiplied by zero is equal to any other number multiplied by zero. Thus, $0 \times 3 = 0 \times 5$, but I cannot strike out the zero from both sides to get that $3 = 5$. This is tantamount to dividing by zero, which is strictly disallowed.

HOW TO PROVE THAT 1 = 2:

Here is an old trick problem often served up to beginners to test their ability to analyze a false premise. We shall prove, using the ordinary operations of algebra, that 1 = 2.

Start with the simple equation

$$a = b$$

Multiply both sides by **a:**

$$a^2 = ab$$

Now subtract b^2 from both sides:

$$a^2 - b^2 = ab - b^2$$

Now factor both sides (you may have to take my word for this, but it is perfectly legitimate. The fallacy is not at this point.)

$$(a - b)(a + b) = (a - b)b$$

Now use the cancellation law to strike out the common factor **(a - b)** from both sides, leaving us with,

$$(a + b) = b$$

Substitute b for a, since they are assumed to be equal from the first equation:

$$(b + b) = b$$

Divide through by b:

$$(1 + 1) = 1$$

or $2 = 1$

Now what went wrong? Simply that we canceled out a factor that was identically zero: **a - b** is zero if **a = b** as we assumed at the beginning. We cannot cancel out a zero factor. Why? I have already showed this:

because any number multiplied by zero is zero. For example, $3 \times 0 = 5 \times 0$, but we cannot strike out the zero factor from both sides to get $3 = 5$. And that is precisely what we did in the above steps.

This is a mathematical example of a false proof that leads to nonsense. There are plenty of Biblical proofs of the same nature; somewhere in his thinking, the scholar has violated some fundamental principle which usually results in total nonsense. Nevertheless, men will almost fight one another over such invalid interpretations. There is something about religious thinking that borders on the fanatic. For some odd reason, men will deliberately close their eyes to glaring inconsistencies in their own Biblical thinking which they would never do in the field of science. I once knew an engineer, a highly qualified one with a degree in his field, that believed some of the most bizarre and fantastic garbage I have ever heard in my life. I could never understand how he could isolate his Biblical thinking from his scientific mentality.

Having said that, I must admit that the Bible does contain some information that science considers fabulous, that is, mythical, or at least legendary, but which some supporters of the Bible claim is true literally. It must be this element that gives rise to fanaticism; the untutored person will fight to keep his thinking unspoiled by science, for he recognizes that some Biblical knowledge can never be supported on scientific principles. I would agree with him if he didn't violate the Bible itself somewhere. One's thinking must never lead to a contradiction of principles stated in the Holy Scripture—to do so is to violate one's assumption that the Bible is inspired of God. Most of those who do think inconsistently about the Bible claim they are guarding themselves against the wiles of Satan. They seem to equate sound, logical reasoning with Satanic wiles.

COMPLETENESS AND CONSISTENCY:

Two philosophical questions remain: is the set of axioms sufficient to prove, or disprove, every possible theorem that can arise in the field of arithmetic? Stated another way, are there statements concerning numbers and their relationship that seem to be true, yet cannot be proven within the framework we have established?

A related question is this: will we ever run into contradictions? Is it possible to display a perfectly valid sequence of logical steps proving a certain theorem, and another perfectly valid sequence that will disprove it?

The first question involves the idea of completeness. As far as I know, it has been answered by the German mathematician, Göedel. No, the system is not complete. There are theorems that, though they can be shown to be true by certain logical arguments, cannot be proven within the domain of arithmetic.

I do not have a good example, but I can show one that has been thought to be of that nature: Fermat's last theorem. Fermat, a famous French jurist and amateur mathematician, proposed the following theorem:

For integers **a, b, c**, and **n,** the equation

$$a^n + b^n = c^n$$

has no solutions for **n** greater than **2**.

This is to be compared with the following:

$$a^2 + b^2 = c^2$$

This equation does have solutions, in fact, there are an infinite number of integer solutions. For example, 3, 4, and 5, or 5, 12, and 13. That is, $3^2 + 4^2 = 5^2 = 25$; likewise, $5^2 + 12^2 = 13^2 = 169$. These numbers, occurring by threes, are called Pythagorean triplets.

In fact, simple algebra can be used to find the set of all solutions, but for $a^3 + b^3 = c^3$ there is no solution, nor is there for any exponent greater than 3. This theorem has been proved for many values of **n**, but it has never been proven for every **n** greater than 2, though countless brilliant minds have worked on the problem since it was first proposed in the 17th century.

If it turns out that this theorem belongs to the class of theorems Göedel discovered, then it may be possible to eventually show that it is true, but one must use additional assumptions besides those we have discussed.

In other words, the arithmetic system of axioms is incomplete. There are mathematical truths forever beyond its pale to establish.

On the other hand, the set of axioms we do have has never been shown to lead to a contradiction. I do not think this has ever been proven; but after millennia of use by some of the world's best minds, it has never yielded a single contradiction. In other words, it is consistent.

Anticipating **PART 3** of this monologue just a bit, let me point out that if the Bible truly is the inspired word of God, then perhaps it represents a logical basis for some truths, but not all. There are things in heaven and on earth that may be forever beyond our ability to prove or disprove based solely on the Bible. However, what we do have is consistent; it has never led to a contradiction. Of course, that is where the devoted Christian parts company with the skeptic. The modern scholar, steeped in this world's skepticism, will point out a great many apparent contradictions in the Bible and scornfully use that to try to discredit the whole idea of Christianity (or Judaism, if one limits himself to the Old Testament.) However, such contradictions may usually be explained in one of several ways: faulty translations, misunderstanding of the original language, copyists' errors, and cultural differences. Finally, many contradictions are the result of misinterpretations, for example, of taking literal what is figurative, or vice versa.

If the Bible is God's infallible word, why did he leave it in such a vague and inconclusive form? Why is it possible to spin out literally thousands of creeds, theologies, and denominations, all claiming to be the true religion with the one and only true gospel or message? Surely, God could have given us a literature that spelled out his intentions and his message in such clear and understandable terms as to prevent all confusion.

Here is an answer that I am satisfied with, an answer that came to me after years of wondering about the problem of so many diverse churches, of so many disputes among those who, from all outer appearances at least, are equally devout, sincere disciples of Christ and of God. I think the key is freedom. God does not want robots for disciples. He does not want to dictate every thought of man. He therefore has laid down only certain guidelines of behavior, with just enough philosophy to help us come up with correct answers in difficult and important cases. It is up to us individually to apply these guidelines and to interpret the philosophy. If we do this with the honest intention of obeying the two great commandments, *Thou shalt love the Lord thy God with all thy heart, and with all thy soul, and with all thy mind,* and *Thou shalt love thy neighbor as thyself,* then God will accept us even if on occasion we have misinterpreted some of the more esoteric portions of Scripture. For it is evident that none of us possess enough "smarts" to understand everything that has been written, and if our salvation depended on having a perfect understanding of all things, we would all be lost.

A corollary to this is that the wicked—and by wicked I mean those who have no desire for the holy things of God or his righteousness, those who would just as soon kill a man as look at him and will do so if it furthers their own selfish ends if they know they will not be caught; those who would rather lie than tell the truth; men who have no intention of repenting of their evil deeds; etc.—the wicked are by their very nature prevented from gaining access to the kingdom of God. If God had spelled out entrance requirements precisely and in minute detail and posed no mysteries that the average person could not easily understand, if even a small child would have no difficulty in discovering the keys to the kingdom, then the wicked could, and would, enter in with the righteous, and immediately begin their plans for usurping authority and taking control, just as they do in this life. In other words, since God cannot lie, if a man showed that he had obeyed every detail of God's plan, God would have to honor his promise and admit that person into his kingdom even though that person still harbors wickedness in his heart.

I am thinking here of two particular cases given in the New Testament: that of Simon, formerly called the Sorcerer, and Ananias and Sapphira. In both cases. they had been admitted into the kingdom, and were considered to be saints. In the first case, Simon tried to convert the power of God into a money making scheme, for which he was severely reprimanded by Peter. Tradition says that he dropped out of the Christian community and began his own brand of gospel, thus establishing one of the very first heresies in Christendom, the *Gnostics*. In the second case, Ananias and Sapphira lied to the Holy Spirit concerning a sale of some property, and pretended to donate the entire sum of money to the church, apparently hoping to gain the respect of the rest of the Christians for being super charitable, but actually secretly keeping back half of the money they earned. Peter pronounced upon both of them the sentence of death. In both cases, they had been admitted into the kingdom of God because they had obeyed his commandments to believe, repent, and be baptized. But their evil nature had not been completely abolished. They soon reverted to their natural inclination with drastic results. God will not permit such wickedness into his kingdom, and will always punish the evil doer..

But because the entrance requirements are vague enough that only one dedicated to God's truth, only one solely interested in righteousness and holiness for its own sake, and only one who truly loves God and his neighbor can enter in; the wicked are automatically excluded.

When I was a small boy, I heard Dad say that all things were possible with God, that nothing for him was impossible. I immediately set to work to invent an impossible situation, even for God. Finally, I proposed this problem: I said, "I'll bet God can't create a door that is both open and closed at the same time." After a lifetime of thinking about it, I have concluded that this is exactly what God has done; Jesus Christ is the door, a door that is both open to the honest and the seeker after righteousness, but closed to the wicked and the hater of all that is good.

Having said all of this, I must hasten to add that this nowise excuses the Christian scholar from making every attempt, from using every mental resource at his disposal, to try to understand God's will and God's plan for himself. In order to do so, he must bring to bear upon the Bible some of the very techniques used by mathematicians in the study of mathematics and science. One of these techniques is the axiomatic approach and the thorough establishment of every truth or every proposed hypothesis by showing that they rest upon the fundamental axioms which themselves must be taken without proof. Fortunately, those axioms are usually self-evident even to fools, so no dissension arises when the proof finally reveals how these axioms support the theorem or the hypothesis in question. Furthermore, if one can show a contradiction arising from the proposed theorem or hypothesis, he is at liberty to discard it with no further waste of time. This is, however, like asking for the moon. There are many Bible scholars who refuse to abandon some view they think they have established, even though it can easily be shown to be totally inconsistent with the precepts laid out in the Scriptures. Since the average person more or less depends on these so-called experts to interpret the Bible to them, they are usually carried away into false doctrine by those experts who refuse to give up faulty thinking. I suspect the real sin here is pride—because of pride, the expert refuses to give in.

Our next part will be concerned with a very useful mathematical function, the exponential. It will be subsequently used to illustrate how pattern matching can be fruitfully employed in Bible study. This is a technique that is woefully under used; it seems that many scholars bind themselves unnecessarily by prejudices and biases and by an undue submission to "the authorities," rather than letting the Scriptures speak for themselves. So when such a pattern is discovered, if it runs somewhat counter to traditional understanding, then the scholar stops right there and refuses to see what is so plainly manifest.

I must hasten to add, however, that we must be balanced in our attempts. The authorities are not always wrong, and the majority opinion is more often right than wrong. All I am saying is that the majority opinion may be wrong in some cases. We must let the Scriptures have the final say. That, by the way, is the Protestant ethic. But I find very few Protestants who make full use of it.

MATHEMATICS AND BIBLE STUDY

Mathematicians are experts at identifying patterns where the unitiated will find only randomness and chaos. It is this ability to see connections and relations between objects, ideas, and concepts that is also of paramount importance to good Bible study. With that in mind, I present the following somewhat elementary development of the exponential function, one of the most useful concepts in not only mathematics but in the natural sciences as well.

PART II: THE EXPONENTIAL FUNCTION

The history of mathematics is as much a history of notation as it is of ideas. Without the proper symbols and rules for manipulation, it would have been impossible for man to have developed some of the profound and eminently useful concepts which we take for granted. Therefore, not only the idea of exponentiation but the development of a suitable means for expressing it must be discussed.

EXPONENTS, EARLY USAGE AND DEFINITION

During the middle ages when the science of algebra was being developed, mathematicians were continually running into expressions that involved the repeated multiplication of a number by itself. For example, 2 X 2 X 2, or 3 X 3 X 3 X 3, etc. Frequently the number itself was unknown so a letter was substituted for it, say, x. The expressions would then look like this: x X x X x or x X x X x X x. Eventually, for brevity, the symbol for multiplication was omitted; we would then have expressions as xxx or xxxx.[1] A polynomial would look like this: xxx + 3xx + 3x + 1. That is, some number, designated by x, is first multiplied by itself 2 times, then one time and also by 3, then only by 3, then the products added to 1.

About this time, another shorthand notation was developed: exponents. Instead of writing the number repeatedly, write it once and indicate how many times it appears in the product with a number placed immediately to its upper right (called a superscript).

Thus, xx becomes x^2 and xxx becomes x^3. The above polynomial then looks like this:

$$x^3 + 3x^2 + 3x + 1$$

Now mathematicians are always on the lookout for patterns and rules. They try to reduce mathematical computations to a set of procedures that anyone can perform, obtaining the solution

[1] We shall also omit the multiplication symbol except when it seems necessary for clarity.

automatically with little or no mental effort.[2] So, as progress was made in algebra, it quickly became apparent that, in addition to producing a neater and more compact notation which was the original goal, a strange byproduct was detected: the exponents behaved according to a set of rules that made them easier to manipulate. For example, if the number 2 is first raised to the second power, then to the third power, and the two products further multiplied together, it is the same thing as raising 2 to the fifth power. First, by our definition of exponents, we have

$$2^2 = 2 \times 2 \text{ and } 2^3 = 2 \times 2 \times 2$$

Therefore,

$$2^2 \times 2^3 = (2 \times 2) \times (2 \times 2 \times 2) =$$

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

But it was quickly noticed that we can obtain exactly the same result by merely adding the exponents:

$$2^2 \times 2^3 = 2^{2+3} = 2^5$$

In case we are dealing with an unknown number, x, exactly the same rule applies:

$$x^2x^3 = x^{2+3} = x^5$$

The reader must be warned that this will not work if we are either talking about two different bases,[3] say $2^2 \times 3^4$ or are adding two terms, say $2^2 + 2^4$. We cannot combine the exponents in such cases. For example, in the above polynomial, we cannot combine the x^3 term with

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2 The fancy name for such procedures is ***algorithms***, a term derived from ***algebra*** itself, which in turn was derived from the Arabian ***al-jabr***.

3 The base is the number itself; the exponent is called the *power*.

the $3x^2$ term because the terms are being added together, not multiplied. Likewise, x^2y^2 cannot be combined because the base for the two terms is different.

> The reader may be tempted to believe that the rule of combining exponents is therefore too restricted to be of much use. Exactly the opposite is true, as will be shown.

Mathematicians soon discovered a parallel rule concerning division of terms with exponents. In the fraction,

$$\frac{x^5}{x^2}$$

it was discovered that the quotient, x^3, could be obtained by simply subtracting the exponent of the denominator from the exponent of the numerator; that is, 5 - 2 = 3, to get the power of the quotient. The justification for this is that one can cancel out two of the x's in both numerator and denominator, leaving a surplus of three x's in the numerator. First, by definition we have,

$$\frac{x^5}{x^2} = \frac{xxxxx}{xx} = \frac{xxxxx}{xx} = \frac{xxx}{1} = \frac{x^3}{1} = x^3$$

Then, by the law of exponents in division, we have:

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

Since we have achieved exactly the same result, our method of subtracting exponents is justified.

LATER EXTENSIONS

We note that so far we are talking only of whole number exponents equal to or greater than 2. Nothing has been said about the use of 1, 0, negative numbers, or fractions. According to the original definition, such exponents are meaningless. It made no sense to speak of a number being multiplied by itself zero times or -2 times for example. Nevertheless, mathematicians, being a curious lot, experimented with these apparently meaningless expressions just to see what they could do with them.

The first extension was very likely the use of 1 for an exponent, such as x^1. This means, according to the original definition of exponents, that the base x is to be used as a factor exactly one time. That, of course, is meaningless. A number is not a factor unless it is being used with at least one other factor to form a product. It is like the only child in a family saying, "I am a brother," when the child has no brothers or sisters. He cannot logically make such a statement.

However, it wasn't long until a way was found for x^1 to make sense. It was noted that when x^3, say, was further multiplied by x, the result was x^4, exactly as though the single factor x had the exponent of 1. Thus,

$$xx^3 = x\,xxx = x^4 \text{ (by definition) } =$$

$$x^1x^3 = x^{1+3} = x^4 \text{ (by law of exponents)}$$

Therefore, a number x with an exponent of 1 (or, as we ordinarily say, raised to the first power), is just the number itself and has no reference directly to being used as a factor in a product. However, as in this example, it can be so used, its implied exponent of 1 capable of being combined with other exponents wherever the law of exponents apply. (We will look at its use in division in a moment.) It is like the only child saying, "I am a brother," looking forward to the day when more children are born into the family.

Let us note another point here, one that has great significance in the entire field of mathematics. In the above example, we obtained the resulting x^4 by two different methods, both starting with the same original expression. Both methods involved the use of the single factor x, the first without an exponent and the second by attaching an implied exponent of 1. Since the results obtained were identical, we have the proof that $x^1 = x$.

Sometimes, a slight variation of this is used. A given expression is simplified by two different but completely valid methods, yielding resulting expressions that appear to be different. Since the original expression is capable of only yielding a single value, we then know that the results, though appearing to be different, are in fact identical. It is the one and only logically correct result masquerading in two different garbs.

See the example in **Part I**, where the expression $e = ab + (-a)(b) + (-a)(-b)$ gave rise to two different appearing expressions, ab or (-a)(-b). Since the same expression was used in both cases, the results must be equal, that is,

$$ab = (-a)(-b)$$

We may state these two closely related techniques as, (1) the result is used to justify the method and (2) the method is used to justify the result. We shall refer to this concept later.

Assuming an exponent of 1 where none appears is also consistent with the rules of exponents in division.

Suppose we have a fraction

$$\frac{x^3}{x^2}$$

If we merely cancel factors we have the quotient x, as follows:

$$\frac{x^3}{x^2} = \frac{xxx}{xx} = \frac{xxx}{xx} = \frac{x}{1} = x \text{ (by definition)}$$

But if we use the rule of subtraction, we obtain

$$\frac{x^3}{x^2} = x^{3-2} = x^1$$

and, as before, $x = x^1$.

Another problem faced by those early mathematicians was this: what does x^0 mean, if anything? If there was a bit of difficulty in making sense of x^1, how much more difficult must this problem be. How can we assign a meaning to the statement, "Use x as a factor no times at all"? The solution to this enigma is to ignore the original definition and look for some meaning within the context of our previous discoveries.

Let us consider the fraction

$$\frac{x^3}{x^3}$$

If we cancel out all common factors in both numerator and denominator, we lose all the x's in both.

$$\frac{x^3}{x^3} = \frac{xxx}{xxx} = \frac{xxx}{xxx} = \frac{1}{1} = 1$$

Or, we could simply recognize that this is a number (in this case, x^3) being divided by itself. In such cases, the answer is always 1 regardless of the value of the expression involved (with the sole exception of zero itself).[4] But, by the law of exponents, we have

$$\frac{x^3}{x^3} = x^{3-3} = x^0$$

Therefore we can make this very positive statement: any number (except zero) raised to a zero power results in the value 1: that is,

$x^0 = 1$ for all x except zero.

We should now summarize what we have discovered about exponents:

1. When multiplying like terms, add the exponents.
2. When dividing like terms, subtract the exponents, denominator from numerator.

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[4] The reason zero is excluded is that division by zero is meaningless. There are occasions when an expression is actually zero in disguise. Raising it to the zeroth power may have a definite value in a few cases, but ordinarily it is to be avoided.

3. Any number raised to the first power is just the number itself.
4. Any number (except zero) raised to the zeroth power has the value 1.
5. Statements 3 and 4 are in total harmony with the first two.

Let us return to the polynomial and use all of our new ideas. We can always place an exponent of 1 on any term without altering its value; likewise, we can always substitute for 1 any non-zero quantity with an exponent of zero. (These two "tricks of the trade" are quite useful in advanced work.) Thus, we will replace x with x^1 and 1 with x^0. The result is

$$x^3 + 3x^2 + 3x + 1 = x^3 + 3x^2 + 3x^1 + x^0.$$

(Note: had the last term been some other constant, say 4, we would merely write it as the product of itself and x^0: thus, $4x^0$.)

Here we notice that x appears in every term with the exponent uniformly decreasing by 1 as we advance from left to right.[5] This is the kind of pattern that mathematicians delight in. Immediately they seek to enlarge the pattern if possible and to increase its scope and power (no pun intended).

By this time, of course, negative numbers had been accepted and the rules of their arithmetic developed. The first obvious question an algebraist would ask is, "Could I extend the above polynomial by adding terms with negative exponents? That is, would the following expression make any sense at all?":

$$x^3 + 3x^2 + 3x^1 + x^0 + x^{-1} + 2x^{-2} + 5x^{-3}$$

---

5 Exponents 1 and 0 are not usually written in expressions except in special cases where it is desired to bring attention to the exponents themselves.

The answer is yes, it makes eminent sense, but we must abandon our original definition of exponentiation almost completely if we are to accept this kind of thing. However, we will justify the use of negative exponents by referring back to that original definition. As we shall see, our new concept is still in perfect harmony with the old--it only extends it into new areas.

Consider once again the quotient of two terms with exponents:

$$\frac{x^2}{x^3}$$

The only difference between this and the first instance we examined is that the fraction has been turned upside down; in the first case, the exponent of the numerator was greater than that of the denominator, while here it is less. When we apply our rule of subtracting exponents, we get -1 for the result; remember, we always subtract the denominator's exponent from the numerator's. We now have the curious expression,

$$\frac{x^2}{x^3} = x^{2-3} = x^{-1}$$

But by resorting to the original definition, we have

$$\frac{x^2}{x^3} = \frac{xx}{xxx} = \frac{xx}{xxx} = \frac{1}{x}$$

We now know further that the x may acquire an exponent of 1 without altering its value. The result then is

$$\frac{1}{x^1}$$

We see that x^{-1} is equivalent to the reciprocal of x^1. Two numbers are reciprocals of each other if their product is 1. For example, 3 and 1/3 are reciprocals because their product, 3 X 1/3, is 1. And what is true for the exponent -1 is true for all negative exponents.

To show how this whole system harmonizes, consider the product of, say, x^2 and its reciprocal x^{-2}.

$$x^2 x^{-2} = x^{2-2} = x^0 = 1$$

so that, indeed, x^2 and x^{-2} are reciprocals.

In general, a negative exponent may be converted to a positive by simply placing 1 over the term to get its reciprocal. (When simplifying algebraic expressions, I like to think of this as removing the minus sign from the exponent and placing it over the base with a 1 above). Thus, we have another law of exponents:

$$x^{-p} = \frac{1}{x^p}$$

Going back once more to our polynomial, we see that those terms with negative exponents may be rewritten as the reciprocals of the same terms with positive exponents:

$$x^3 + 3x^2 + 3x^1 + x^0 + x^{-1} + 2x^{-2} + 5x^{-3} =$$

$$x^3 + 3x^2 + 3x^1 + x^0 + \frac{1}{x^1} + \frac{2}{x^2} + \frac{5}{x^3}$$

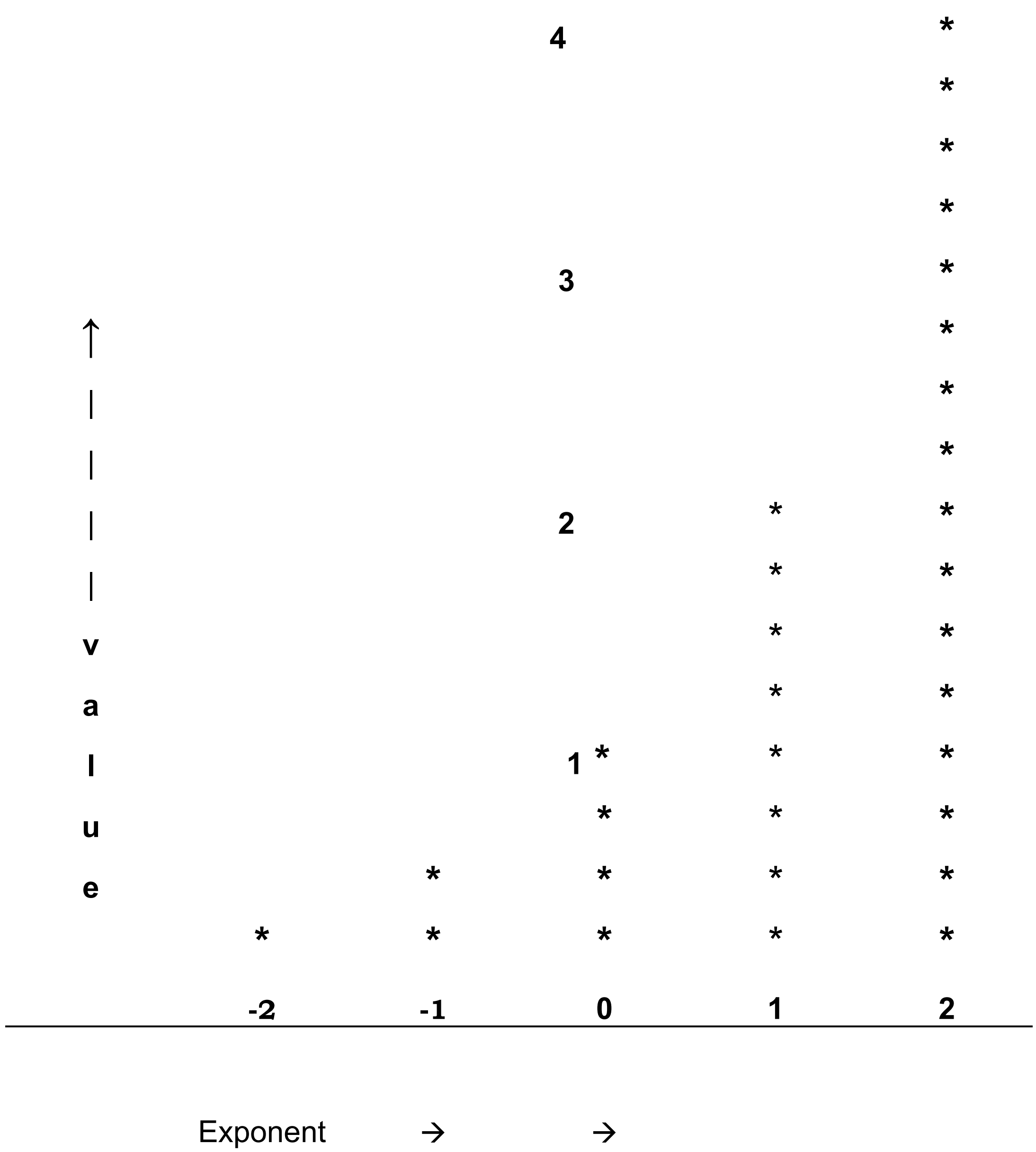
At this point, we must recognize that we are no longer very much dependent on our original definition of exponents. We have discovered a higher law for exponents that permits greater scope and, shall we say it, reveals far more amazing and wondrous ideas that were totally unsuspected by that first mathematician who toyed with using a number to indicate how many times he was multiplying a quantity by itself.

GRAPHICAL EXTENSIONS

We shall draw a graph to illustrate what we have learned. To make it as simple as possible, let us use a known base, 2, and plot its value raised to the powers of -2 up through +2. To that end, let us prepare a table showing the powers of two in the left column, the algebraic expressions in the middle column, and their values in the right. Begin with -2 and work upwards to +2:

Exponent	Expression	Value
-2	2^{-2}	$1/2^2 = 1/4$
-1	2^{-1}	$1/2^1 = 1/2$
0	2^0	1
1	2^1	2
2	2^2	4

Graph of table: the value of the term is plotted on the vertical axis versus the value of the exponent on the horizontal axis. Each vertical division represents 1/4 and each horizontal, 1/10.



The graph shows only those points for which we have information at this time. But we notice that the points appear to lie on a curve that sweeps upward in an ever increasing curvature. Note that at this time, nothing is known about exponents that fall between those given, say 1/2, for example.

Once again, a mathematician's curiosity is aroused: he asks the following questions: are these points really lying on a curve as they seem to be? If so, what will be the intermediate values to those given? What, for example, is the value of $2^{1/2}$?

By carefully drawing a smooth curve through the plotted points on accurate graph paper, we obtain a curve that looks something like this:

```
           4.00                *
              |
              |
              |
           3.00
              |             *
              |
              |
           2.00       *
              |
           1.41Y    *
              |
           1.00*
              |
       *      |
 *            |
-----------------X----------------------------
-2     -1     0   .5  1      2
```

Now examine the point corresponding to an exponent of 1/2 (= .5). If we have drawn very carefully, we should be able to read off values to the nearest one hundredth. For $2^{1/2}$, we find the value on the vertical axis of 1.41, very nearly. The observant mathematician will at once exclaim, "Why, that is pretty close to the square root of 2!". Which it is. In fact, it is precisely the square root of two (even though we can't read it exactly). Our justification for this comes about as follows. Construct the expression

$$2^{1/2} \text{ X } 2^{1/2}$$

which is a number times itself. We do not yet know what that number is but we do know that it must be the square root of something; for when a number is multiplied by itself, we say it is squared, and, by definition, the number being squared is a square root.[6]

By using our property of exponents, we can obtain the square of this number by merely adding the exponents and applying to the given base 2. Thus, we have that

$$2^{1/2} \text{ X } 2^{1/2} = 2^{1/2 + 1/2} = 2^1 = 2$$

That is, since the number $2^{1/2}$ when squared gives 2, the number itself must be the square root of 2.

By similar reasoning, we can show that $2^{1/3}$ is the cube root of 2. In general,

$$x^{1/p} = \text{pth root of x.}$$

We can raise this number to a power just like any other. Thus, $2^{1/2}$ raised to the 3rd power would look like this:

$$(2^{1/2)3}$$

[6] This, of course, is a geometric concept. Given a side x, then x^2, read *x squared*, is the area of a square whose side is x. Inversely, given the area of a square, its side is called the square root of the area.

It is relatively easy to establish that this will result in $2^{3/2}$, which is gotten by multiplying the two exponents together. Thus, in general,

$(x^p)^q = x^{pq}$ or

$(x^{1/p})^q = x^{q/p}$ since $(1/p)q = q/p$

For example, $(3^2)^5 = 3^{10}$ and $(3^{1/2})^5 = 3^{5/2}$

To see that this is sensible, consider $(2^3)^2$. The exponent 2 means to write the expression in parenthesis down twice and multiply together: $2^3 X 2^3 = 2^{3+3} = 2^6$. Here we added the exponents to get 6, but we could have more easily obtained it by multiplying the two exponents together.

Now back to our original problem with fractional exponents: the hard part was determining what the fractional exponent 1/p meant. Now that we know it means the pth root of the base, we may extend this to a fraction with any numerator by noting that the denominator tells us what root we are taking and the numerator tells us what power we are raising it to. Hence,

$2^{3/2} = (2^{1/2})^3 = 1.414^3 = 2.828$ approximately.

That is, we are raising the square root of 2 to the third power.

Now for the amazing thing. If we inspect our graph above, we will note that the value 2.828 is indeed the value we could have read from the curve corresponding to an exponent of 3/2.

Why is this amazing? It is because here is a curve that was drawn by intuition, based on a very few discrete points that fulfilled our original definition of exponents or were an easy extension of that definition. Now we see that this curve can account for infinitely many more exponential values--the fractions--which were not related directly to our original definition. It can further be shown (though I will not do so) that there are other points not yet accounted for lying on the same curve, and that there are infinitely many more of these than there are fractions.[7] For one simple example, the value PI, which is the ratio of a circle's circumference to its diameter and approximately equal to 3.14159, also has its corresponding point on the curve. This number cannot be written as a fraction exactly. Nevertheless, it may be used as an exponent and obeys all the laws pertaining to them.

This curve—which goes by the name of an exponential function--is simply and positively amazing. When we first began using exponents, we had no idea that there was such a curve. All we had were a few discrete points. For all we knew, no such curve may have even existed, that is, there might not be any points in between those we already had. And even if there was a curve, it might not proceed smoothly as this one does. It might have been filled with infinitely sharp spikes or bottomless holes; it might have gaps in it; it might rise for a while, then descend. But the function we constructed does none of these things; it accounts equally well for all of our originally defined points as well as those strange ones that were introduced by clever tricks; and it permits them to be manipulated among themselves by the simple rules of arithmetic, always obtaining a result that is in total harmony with the rest. It is, I repeat, one of the most amazing things in all the realm of human thinking.

7 For those with a mathematical background, I am thinking of the irrationals which we have not discussed. They are infinitely more numerous than the rationals (fractions) themselves.

What we have done here is of utmost importance: we began with a very primitive concept; then we have watched it grow into areas for which we had no direct understanding, areas that, as a matter of fact, appeared to be utter nonsense, and then find that this new system not only accounts for all of the old knowledge, but very elegantly encompasses new ideas that were entirely unsuspected, making good sense of the nonsensical. Unless mathematicians had stumbled onto these elegant concepts, exponentiation would have remained a very trivial tool of little use beyond making notation a bit neater. Those expressions not directly included in the definition, such as $3^{1.414}$, would have remained meaningless chicken tracks on a madman's manuscript.

How did this come about? Was it a pure accident, or was man guided by some mystical force, perhaps of a supernatural nature? There are proponents of both views. For myself, I assume this kind of knowledge was what man stole from the Garden of Eden. It can be used for both good and evil, and a little further on I intend to show how some of this kind of thinking can be used for studying the holy things of God written in the Bible.[8]

Back to the curve: it reveals at once the following useful and powerful relationship concerning the powers of 2. For any two given powers of 2, the one with the greater exponent will have the greater value: that is, $2^{3.1}$ is greater than 2^{3}. I realize this may not startle the

[8] I am not calling the study of mathematics evil; but neither could it be said to be particularly holy and sanctified either. Many fundamentalists look with disfavor on higher learning, but I am convinced that its principles can be used for the Kingdom's work.

reader—he would probably have guessed as much since it seems so sensible. So let me put it another way: given three powers of 2, the one with the least exponent will lie to the left and nearest the bottom of the graph, and the one with the greatest exponent with lie higher and to the right, while the one with the mid-value exponent, lying as it does between the other two, will plot as a point between the others, not only horizontally but vertically.[9] In other words, the point will lie on the curve and not be found somewhere else.

Here is an example of how the principle can be used. We wish to determine an approximate value for $2^{3.14159}$. Note that the exponent lies between 3 and 4. Therefore, the result will lie between 2^3 and 2^4. The first power of two is equal to 8, and the second, 16. Thus, our result will lie between 8 and 16, closer to 8. Without a simple test like that, we would be hopelessly in the dark about what kind of result we should expect. Further, if we go ahead and calculate the result using any of various techniques that are somewhat involved, we have a good check on the correctness of our answer. I will use a pocket calculator: $2^{3.14159}$ = 8.82496, which indeed does lie between 8 and 16 but closer to 8.

A curve such as we are investigating goes by the name of "a continuous, monotonically increasing, real valued function." This is saying, principally, that the curve rises uniformly upward as we traverse it left to right, with no jagged and unexpected spikes, no crevasses or

[9] If you who are reading this are a mathematician, please suppress your laughter; of course, I am not interested at this point in complete rigor or in stating these things in absolutely precise terms. I think that such language stifles the curiosity of many who otherwise are quite capable of seeing the relationships. We are, after all, only trying to illustrate another principle to be expressed a bit later.

holes, no hills, and no valleys. It is increasing in value in an extremely smooth and continuous fashion.[10]

EXTENSIONS INTO THE COMPLEX PLANE

There is yet one more extension of the idea of exponentiation: exponents may also be complex numbers. These are numbers that involve the square root of minus one. There isn't any ordinary number, positive or negative, that is the square root of minus one. So mathematicians decided to assign it a symbol, usually the letter *i*, which stands for *imaginary*. When combined with an ordinary number (called a *real*), we have complex quantities that, with the arithmetic operations suitably defined, can be added, multiplied, subtracted or divided like the numbers of ordinary arithmetic. In particular, both complex and real numbers may be used as exponents on either complex or real bases. There is almost no connection at this point with our original definition of exponentiation, yet some of the finest and most elegant mathematics with the greatest practical value have come from this source. These quantities appear abundantly in all of the natural sciences; technology draws heavily upon them for its techniques of design and production of a good portion of contemporary goods, especially in the field of electronics and electrical power.

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[10] For the mathematically educated reader: there is one restriction that ought to be mentioned. What we have said is true for values of the base greater than 1. Exponential functions are defined for other values, such as negative bases, but the functions are such that sometimes curves do not exist.

Though complex numbers are among my favorite forms of mathematical quantities, I must desist discussing them any further. You will have to take my word for it that it is in this area that the principles we are trying to establish are most vividly and forcibly manifested. More than this would carry us too far afield (pun intended).[11]

LOGARITHMS

One early example of using the laws of exponents for practical matters is the concept of logarithms that was developed by the Scottish mathematician Napier. Every school child who has taken algebra becomes acquainted with tables of logarithms, usually by being assigned the task of doing some multiplication with them. Not really understanding what he is doing, the student usually grows to detest them with a passion.

To see the connection between logarithms and exponentiation, consider this function[12]:

$$x^p = y$$

There are three quantities here: x, p, and y. Our problem up to this point can be stated as follows: given the first two of these, calculate the third. That is, given a base, x, and any number for an exponent, p, what

11 A *field* is a mathematical concept. The ordinary numbers of arithmetic form a field. So do the complex numbers.

12 Up to this point I have resisted calling expressions by their mathematical name, but for those who may be mildly interested, substitute ***function*** for ***expression***.

is the value, if any, of the function y? For example, what is the value of $y = 2^{3.1}$?

The concept of a logarithm of a number is similar except that we are given the first and the third of these quantities and seek to calculate the second: given the base, x, and the value of the function, y, we wish to establish what exponent p we must use to obtain the function. For example, given

$$2^p = 2.828$$

what is the value of p which will make this true?

Tables have been constructed for obtaining approximate values for the unknown exponent.[13] Where for simplicity I have been using 2 for the base, the tables use either 10 or the number 2.71828 (approx.), called *e*. (We shall shortly show why this strange looking decimal is used.) It is relatively easy to go from one base to another so that in reality only one table is needed.

By looking up the exponents, called logarithms for some unknown (to me) reason,[14] of two given numbers, their product can be obtained by simply adding the logs and looking up in the table to see what number goes with this value (called the *antilog*). They may also be used to raise a

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13 If some reader should express curiosity as to how these tables are prepared, I can only say it is a problem of the calculus. Certain functions, called infinite series, can be constructed that one can use to calculate to any degree of precision the numbers in a logarithm table. These techniques are commonly built into a computer, either as a software or as a hardwired program. The programmer can call upon these built in functions to calculate the logarithm at will.

14 The dictionary says it comes from the prefix log-, meaning "word" or "thought," from the Greek ***logos*** and the Greek ***arithmos*** meaning "number" and from which we get the word "arithmetic." It looks a little contrived to me.

number to an arbitrary power: look up the log of the number, multiply by the power, then look up the antilog of the result. In the days before computers, these were about the only shortcuts that scientists had for doing lengthy calculations and are about the only use high school students ever make of logarithms.

However, logarithms, exponentials, and their relatives are at the very heart of natural science. I will later give a few examples before passing on to the main theme of this essay.

THE NUMBER e

There is a number that cropped up early in mathematical investigations that appears throughout the whole field of practical and theoretical science. Since it is intimately associated with exponentiation, I want to look here briefly at its definition and to calculate its value approximately. This will give us a good opportunity to look at an important concept, that of limits. Then I wish to show just as simply as possible why this number is so important.

The number is usually symbolized by *e* which stands for exponential. We are going to show why this number is important and also how to calculate it. This calls for a little banking expertise.

Banks loan money, or pay depositors, according to two basic interest schemes: simple and compound. Let us use this notation:

P = principal, the original money deposited or loaned.

r = rate of interest, expressed as a decimal.

t = time of deposit or loan in periods, usually years.

I = interest earned or due.

A= amount earned or due; it is the sum of the principal and interest earned.

We have the following formula for simple interest invested at the rate r for time t:

$$I = Prt$$

For example, if we invest \$1000 at 10% for 1 year, we have:

$$I = 1000 \times .10 \times 1 = 100$$

For 2 years:

$$I = 1000 \times .10 \times 2 = 200$$

The amount A due at the end of the time t is simply the sum of the principal and the interest earned:

$$A = P + I = P + Prt = P\,(1 + rt)$$

In our example, for 1 year, then for 2 years, we have:

$$A = 1000\,(1 + .10 \times 1) = 1100 \quad \text{for 1 year}$$

$$A = 1000\,(1 + .10 \times 2) = 1200 \quad \text{for 2 years}$$

However, to remain competitive, banks usually offer their depositors compound interest. This means that the interest earned at the end of a period is reinvested at the same rate, thus earning a bit more.

The formulae are a bit different, too. The interest at end of one year is still:

$$I = Prt$$

But for the second year, this interest is also invested. Without going into all the algebra, here are the formulae:

$$A = P(1 + r)^t$$

$$I = A - P$$

(We have to reverse the procedure in this case; calculate the amount due first, then subtract the principal to get the interest.)

Note that the t has moved from inside the parentheses as a multiplier of r to become an exponent. This little change has enormous implications, far beyond what one might suspect. First, however, let us calculate the amount due at the end of the first year:

$$A = 1000\ (1 + .10)^1 = 1000 \text{ X } 1.1 = 1100$$

which is exactly the same as for simple interest.

Let us now calculate the amount due at end of two years with interest compounded annually:

$$A = 1000\ (1 + .10)^2 = 1000 \text{ X } 1.1^2 = 1000 \text{ X } 1.21$$

$$= 1210$$

As you can see, the amount due at end of two years is slightly more, in fact, $10 more, than for simple interest. The extra $10 is just the interest earned during the second year on the interest earned during the first year.

So far, this is standard procedure for certificates of deposit and loans. But a lot of people have only checking accounts with banks. Even so, they may still profit by compound interest. In order to attract customers, banks will pay the depositor a rate on his average balance compounded periodically, a rate that is less than they earn themselves on the same money. (Banks will calculate an average total balance for all customers and an average daily withdrawal, then skim off the difference to invest for their own profit. The danger, of course, is that if everyone should withdraw their total balance at once, the bank would not have sufficient funds to satisfy. But, that is banking practice.) I think that in former times this was calculated as simple interest, but competition has driven them to offer compound interest schemes. Some do this on a semiannual basis, some quarterly, some monthly, and some even daily.

Now it begins to get interesting. Let us modify the above formula to calculate interest and amounts for compounding on periods less than a year. The first thing to note is that the rate is always stated as the nominal annual rate. For periods less than a year, the annual rate is slashed accordingly. For example, our 10% will become 5% for semiannual, 2.5% for quarterly, .833% for monthly, and .0274% for daily compounding. That is,

$i = r/n$ where n is the frequency of compounding.

Adjusting our formulas accordingly, we have:

$$A = P\,(1 + r/n)^{n}$$

(here, A is the amount due at year's end)

Now let us calculate how much we will earn in a year's time if our bank compounds for each of these intervals. In the following calculations, the number in parentheses at the left stands for the frequency of compounding: 2 for semiannually, 4 for quarterly, 12 for monthly, and 365 for daily. For clarity, let us look only at the parenthetical quantity since it is the only thing that changes. That is, assume the principal is $1; to get the true dollar amount, merely multiply the result by the actual principal--in our case, $1000:

(2) $A = (1 + .10/2)^2 = (1.05)^2 = 1.10250$

(4) $A = (1 + .10/4)^4 = (1.025)^4 = 1.10381$

(12) $A = (1 + .10/12)^{12} = (1.00833)^{12} = 1.10471$

(365) $A = (1 + .10/365)^{365} = (1.000274)^{365} = 1.10515$

Notice how the quantity inside the parenthesis is growing smaller but never quite reaching the value 1. If the quantity ever does reach 1, then no matter how large an exponent we use, we will end up with 1 because 1 raised to any positive power remains exactly 1. But we know that it will never quite reach 1 because we are adding a fraction to 1, which, though extremely small, is nonetheless a finite amount. On the other hand, the exponent is growing larger and we know that a number larger than 1, no matter how little the difference, raised to a sufficiently large power will become enormous.

We are most interested in this conflict of two forces, one trying to keep the quantity small and the other trying to make it large. Which one will win? Will the number remain small, in the neighborhood of 1, say, or will it continue getting larger without bound?

Before we answer these questions, let us look closely at the results. Multiplying each by 1000 gives us the following sequence of dollar amounts:

1102.50, 1103.81, 1104.71, and 1105.15.

Notice that, though compounding daily does increase the interest earned, it does so only slightly more than, say, monthly. Most people, I suspect, think that compounding daily is the sure way to riches. But it is really only a gimmick used by banks to acquire more business, and, even though the increase is small, they usually lower their nominal yearly rate sufficiently to offset most of the gain.

We might ask, what would be the value if interest were compounded every second of the day for a year? I will give an approximate answer here using my pocket calculator[15]: $1105.17. That is, it only increased 2 cents over that of daily compounding!

But we are truly interested (no pun intended) in $(1 + r/n)^n$. I am not sure where this expression first reared its ugly head, though I strongly suspect it was in the banking business. Mathematicians quickly grabbed on to it, made a small change, and saw it appear in various and sundry places in the natural sciences that had absolutely nothing to do with banking. Here is the change they made: substitute 1 for r. We then have this expression:

$$(1 + 1/n)^n$$

The first thing we wish to ask of it is what happens when n is increased indefinitely. That is, how does the expression behave as n is increased enormously, for example, to 10,000,000,000? To get an idea let us make a table of values for the expression for a selection of n beginning with 1 and increasing through a few powers of ten:

[15] As a matter of fact, I used the calculator for all of these computations. One could, of course, use ordinary arithmetic, but raising a quantity to a power of 365 or even 12 would prove extremely laborious. Even so, I had to make use of a special formula to get this last figure, since even the calculator had insufficient capacity to evaluate the above expression for the number of seconds in a year.

n	(1 + 1/n)		Value
1	$(1 + 1/1)^{1}$	$= 2^{1}$	2.000
10	$(1 + 1/10)^{10}$	$= 1.1^{10}$	2.594
100	$(1 + 1/100)^{100}$	$= 1.01^{100}$	2.705
1000	$(1 + 1/1000)^{1000}$	$= 1.001^{1000}$	2.717
10000	$(1 + 1/10000)^{10000}$	$= 1.0001^{10000}$	2.718

Notice how the result keeps increasing but not by a whole lot. I am going to make some statements here that you will have to accept on authority. This sequence of numbers is said to be approaching a limit. Which is to say, they will not grow infinitely great. We can say it this way: there exists a number which will always be larger than or equal to all members of the sequence. In this case, one could guess that would be true for 5, say, or even 4, or, in fact, any number equal to or greater than 3; but the limit has this additional property: it is the *smallest* such number. Any number less than the limit, no matter by how small an amount, will eventually be exceeded at some point in the sequence. To repeat, this number is called the limit of the sequence. It is one of the most important concepts in the whole realm of mathematics.

To make the concept as simple as possible, look at each of the numbers in the above table closely. Start with the third row to give the process time to stabilize. For n = 100, we can bound our sequence by 2.7 and 2.8. That is, no further refinements will ever increase our number beyond 2.8 nor reduce it below 2.7. For n = 1000, our bounds become a little more precise: 2.71 and 2.72. Finally, for n = 10000, the lower bound is 2.718 and the upper bound is 2.719. We know (I am not proving it, however) that the limit will be greater than 2.718 but less than 2.719. As we keep adding members to the sequence, resulting in more decimal places of precision, we continue squeezing the limit between bounds to get a more accurate representation.

However, in this case, as is true for a large number of interesting mathematical quantities, we can never finish the process; we can never write down the quantity exactly with a finite number of decimals. The number is said to be irrational, that is, cannot be expressed as the "ratio" of two whole numbers. (Decimal fractions are just a class of rationals whose denominators are powers of ten.) Irrationals, then, require an infinitely long string of decimals for their representation, which simply means that we cannot do it. Maybe God can.

Mathematicians have named this particular irrational *e* and have calculated its value (by other methods) out to several decimal places for huge n. It is approximately

$$e = 2.71828\ 18284\ 59045\ 23536\ \ldots.$$

That is, e lies between 2.71828 and 2.71829. As we go to the next decimal place, the number has increased past 2.71828 but will never reach 2.71829. Or, we could go out a few more places and say that e lies between

2.71828 18284 59045 23536 and 2.71828 18284 59045 23537

Mathematicians can prove that as we calculate any further precision, it will increase our number past the left hand quantity (the lower bound) but never to the extent as to reach the right hand quantity (the upper bound). This process is said to be *convergent*, and the series of quantities are converging to a limit. In the limit, the greatest lower and the least upper bounds coincide. That limit, I repeat, is the number *e*, the base of natural logarithms.

> The mathematical definition of e, then, is this: it is the limit of the expression $(1 + 1/n)^n$ as n approaches infinity. In shorthand notation, it becomes:
>
> $$e = \lim_{n \to \infty} (1 + 1/n)^n$$

Now to finish our banking lesson. When banks state a nominal yearly rate r, we can calculate the effective rate R for compounding at every instant. That is, for every infinitesimal moment of time, we reinvest all that has been earned up to that point including our principal. (Every second will be close enough.) It will become:

$$R = e^r - 1$$

For example, given our fabulously large rate of 10%, if our bank compounded every second of the day for a whole year, we could calculate our effective rate as $e^{.10} - 1$, which computes to 10.517%. This is not a startling increase, and banks could easily afford to offer it. However, they would be pestered by customers asking how they could calculate their interest, so the banks usually stop at daily compounding. Even here, it is not a simple task to calculate, but the process can be explained to the average person.

When we first defined compound interest, it was for whole numbers of years. Now we have developed the formulae for fractions of a year. Both can be put together into one pair of formulae:

$$A = P e^{rt} \quad \text{and } I = A - P$$

where t can be any number greater than zero, fractions included.

For example, the principal of $1000 invested at 10% compounded instantaneously (or every second) for 2 years and 219 days (2.6 years) is:

$$A = 1000\ e^{2.6 \times .10} = 1296.93.$$

$$I = 1296.93 - 1000 = 296.93.$$

However, banks do not, as far as I know, compound every instant or even every second. What we have here is a solution looking for a problem. Fortunately, we won't have far to look; there exists a multitude of natural processes that do make use of this formula.

THE COMPOUND INTEREST LAW

The whole principle behind compound interest is that the rate of increase of money is proportional to the amount of money on hand. The more money we accumulate, the faster it increases. This is a fundamental law of many natural processes. I will list a few below. The law has, of course, been restated by mathematicians in their elegant language of differential equations, but I promised to stay out of calculus in this essay. I will only say that differential equations describe a multitude of natural phenomena, with one of the ever present problems of applied mathematics being to solve all sorts of differential equations as they arise in research. Oddly enough, it is relatively easy to write down a differential equation that describes some process, but it may prove to be extremely difficult or even impossible to solve. However, the little rascal we have been concerned with, the one that describes the rate of growth of a quantity as proportional to the magnitude of the quantity, is a very simple one that has been understood for centuries. We hasten on.

The compound interest law may be stated as follows:

$$Q = q\ e^{rt}$$

Here, I have substituted Q for A and q for P to avoid confusing it with compound interest on money. The lower case q stands for any quantity at the beginning of the process—often called the "initial value"—and Q for the quantity at the end of the interval t. The time t may be stated in whatever units is reasonable, including fractions of units. Thus, t = 2.36 could be used.

This mathematical expression is called "the exponential function." Many functions are exponential functions, but when one says "the exponential function," this is the one that is meant. The letter *e* is just an abbreviation for "exponential."

APPLICATIONS

Plants grow at an exponential rate. This should be clear when we realize that each cell has the ability to reproduce itself. Hence, the larger the plant, the more cells there are and the faster it increases. It may not look like a fast increase to the casual eye, but remember how small an acorn is and how large an oak tree can grow. The growth may be spread over centuries, simply because the rate is fairly small. (The same thing is true for animal life with this difference: growth does tend to slow down with maturity.) Of course, there are complicating factors, such as weather, soil condition, crowding, etc. But the compound interest law is the predominant law regulating the growth of a plant.

Population tends to grow according to this law as well. Once again, this ought to be obvious when we understand that every human couple has the ability to reproduce. The more there are, the faster the population grows. As before, there are other factors to be considered, such as war, epidemics, natural disasters, etc. But under normal circumstances, a population grows at an exponential rate.

Let me hasten into the field of atomic physics. It is here that these formulae are most precise. Because we are dealing with absolutely enormous numbers of particles, the "instantaneous compounding" property comes into play.

The process of radioactive decay, for example, very closely approximates an exponential function. There is one difference, however, from the foregoing examples. The exponent is negative instead of positive. That is,

$$Q = q\ e^{-rt}$$

What is involved here is rather simply stated, though the actual mechanism still baffles the physicists. For certain elements, the nucleus of the atom is unstable. It has a tendency to "explode," so to speak, and break up into pieces which are smaller atoms of lighter elements, atomic fragments, and the emission of radiation. Apparently, atoms spontaneously decay in a random fashion. We cannot determine when a particular atom will decay, but given the vast quantity of atoms available, there will be a certain proportion of them decaying at any particular moment. After they have decayed, there are fewer atoms of the original element remaining, but exactly the same proportion of those remaining will decay the next instant. So here is a case where the quantity ***decreases*** rather than increases at a rate proportional to the number on hand at any one moment.

The technique of radioactive carbon 14 dating of fossils depends on this notion. There are two additional assumptions upon which the validity of the whole scheme stands. One is that the generation of radioactive carbon 14 by natural processes is a constant and has remained so throughout millions of years. The other assumption is that living matter will absorb some fraction of carbon 14 into its system and that this fraction remains constant throughout the life of the organism. That is, as the radioactive carbon decays, more is ingested by the organism so that the total amount remains constant. At death, however, the organism can no longer replace the atoms that have decayed. After an interval from death, the number of atoms remaining of the original quantity can be measured, and the time from death determined.[16]

Radioactive decay is also a fundamental notion in the whole field of atomic and nuclear physics. The composition and life cycle of the sun and stars intimately involves this concept. There is also the inverse process of fusion in which atomic particles are fused together instead of divided. This process, too, involves exponential functions.

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[16] I have no quarrel with the general principles involved here. But it is my opinion that one or both of the extra assumptions may be faulty. I have not yet completely reconciled myself to the scientific conclusion that certain fossils have been determined to be millions of years old. Perhaps, but maybe not.

Another field that makes heavy use of exponential functions is statistics. One of the most common forms of frequency distributions is the Gaussian distribution,[17] also called the normal or the bell shaped distribution. It is described with the following general form:

$$F(x) = k\ e^{-x^2/2}$$

It is difficult to show this with my printer, but the exponent on e is the entire quantity

$$\frac{-x^2}{2}$$

which itself involves an exponent. Here is an example of an exponent inside an exponent.

This mathematical function describes so many phenomena appearing in nature that it is impossible to do it justice. The distribution of vital statistics of a population, such as height and weight for example, follow this law. Scores on intelligence tests also follow the law. Weather statistics, such as rainfall from year to year, accumulation of snow, temperature fluctuations, etc. all can be described in terms of the Gaussian distribution. Various errors, such as the tolerance in the dimension of an object being machined, often follow the normal distribution.

There are other probability distributions, as they are called, that also partake of exponential functions of one kind or another.

[17] Named for Carl Frederick Gauss, a famous German mathematician.

Another example, and one that we all are familiar with, is the sag of a cable or a string with its ends fastened to two uprights, such as electrical power wires or telephone cables, or even a clothesline. The general form of this curve, called a *catenary*, is given by

$$y = c\,\frac{(e^{x/c} + e^{-x/c})}{2}$$

The quantity inside the parenthesis along with the denominator 2 is called the hyperbolic cosine of x/c. Here is another area that I must resist getting into, but the analogy of the hyperbolic functions with the circular functions is absolutely amazing. All we wish to point out here is that this common curve can be described with exponentials.

SOME REASONS FOR THE IMPORTANCE OF THE EXPONENTIAL

We have already stated one reason for the importance of the exponential function. Many natural processes obey the compound interest law in that the rate of increase (or decrease) of a quantity is directly proportional to the quantity in existence at that moment. As it increases (or decreases), the rate of its increase (or decrease) changes in proportion.

This is all related to an even more fundamental statement. Of all the infinitely many functions that could be studied, the exponential function is the only one whose derivative is equal to itself.[18] Since the rate of

[18] This, of course, is a concept of calculus, of which I have chosen to say nothing.

increase is the derivative of a function with respect to time, then we are saying that, of all functions, the exponential is the only one whose rate of increase is just itself.

However, there are some other functions that come close to this "ideal." The trigonometric, or circular, functions of sine and cosine also show this characteristic. The derivative of the sine is the cosine, and the derivative of the cosine is the negative of the sine. Now if you were to plot both curves accurately on graph paper and cut one of them out, it could be repositioned to overlay the other one exactly. In other words, they have exactly the same shape and size, differing only in their position on the plot. (They are displaced horizontally by an amount that translates into a circular measure of 90 degrees.) So here are a class of functions that behave almost like the exponential in that the derivative of themselves is (almost) themselves back again. Is there then some relationship between the trigonometric functions and the exponential?

Yes. In fact, the sine and cosine functions can be described as

$$\sin x = \frac{(e^{ix} - e^{-ix})}{2i}$$

$$\cos x = \frac{(e^{ix} + e^{-ix})}{2}$$

where *i* is the imaginary number, the square root of minus one.

All radiation, be it sound waves or radio waves, light waves, infra-red, ultra-violet, x-rays, or gamma rays, plus many other similar kinds of phenomena, can be described as some combination of the trigonometric functions. Hence, the property of a function to preserve itself upon differentiation pervades the entire universe in a multitude of ways. For our discussion, we only note that these functions may be rewritten in terms of our exponential function. (In order to do so, however, we were forced to introduce that obscure and cryptic little quantity, the square root of minus one.)

I shall not pursue this line of thinking any further since it would require literally volumes and a much better mathematician than myself to do so.

SUMMARY

Exponents, introduced at a very primitive level and whose only purpose was in improving the shorthand notation used by mathematicians, ultimately took on a life of their own. The concept of exponentiation invaded the entire realm of natural science. Its properties, while reminiscent of the early definition, go far beyond what anyone dreamed possible in those early formative years.

I ask once more, was this pure, blind, staggering luck, or was man guided by some kind of a spiritual (and supernatural) force, the product of a mind far surpassing his own?

MATHEMATICS AND BIBLE STUDY

What this essay is NOT about: I do not intend to delve into numerology or to attempt to define symbolic properties for the numbers that appear throughout the Holy Scriptures, with the possible exception of the number seven. I have read literature on this subject but have always come away feeling let down, or that I had really learned nothing at all from their usually fantastic conclusions pertaining to the spiritual meaning of numbers. My essay will involve genuine Scriptural analysis of the issues involved, with the greatest effort expended in trying to throw some light on several relatively obscure passages, and to make sense of what some might think of as contradictions or spurious writings.

In short, I want to present a case for the actual inspiration of the Bible, written as it was by men filled with God's Holy Spirit, and not by politicians who were trying to organize a political party aimed at revolution and designed to give them power and leadership, a concept particularly favored amongst liberal circles. Actually, no proof can be given for many of the ideas expressed herein except that they present a consistent and a harmonious picture of God's dealings with the human race, particularly his great mercy and loving kindness and his efforts to perfect man and to lead him to salvation.

PART III: BIBLE STUDY —USING THE FOREGOING PRINCIPLES

We first must establish our "axioms" on which we base our Biblical studies. I shall not try to be totally complete with this, but merely offer enough examples to show what I mean.

The very first, and really, the most important, is

(1) There is an almighty God, the creator of the total universe.

We could not discuss anything about the Bible being God's inspired Word without accepting that He exists. The second axiom is as important as the first:

The second axiom derives from this first:

(2) The Holy Bible, delivered to us by Israel, is God's inspired word, containing instruction and commands for us to obey.

The Bible, both Old and New Testaments, is God's inspired word.

On this score, I truly fail to comprehend how some scholars can spend their entire professional career as theologians without believing axiom (2). Nevertheless, I have met individuals, pastors of large congregations, who deny the inspiration of the Holy Bible, the divinity of Christ, the miracles, and the stated purpose of the Bible which is to save men from eternal damnation by leading them to Christ.

Another axiom is this:

(3) God will not, indeed cannot, lie. Whatever he says will come to pass, one way or another.

We are told this by the writer of the epistle to the Hebrews, chapter 6, verses 17-18. Truly, we can depend upon everything God says, which is comforting to the righteous but is extremely threatening to the wicked. Usually, the wicked choose not to believe that God means everything he says, or else they don't believe he said it.

As pertains to Bible study itself, there is an axiom that is equivalent to one I emphasized in Part I: Equivalent ideas, concepts, and phrases may be freely substituted for each other wherever they occur. To illustrate this point, let me comment upon an expression found frequently in the Scriptures: *the fatherless.* For years I suspected it had a richer meaning than the literal one, that of a child who has lost his father through death of some other means. In practically every place the phrase occurs, we are told to love and comfort the fatherless. Recently, the truth of the saying dawned on me. We, the Gentiles, are the fatherless, for we cannot claim Abraham, Isaac, and Jacob as our physical, biological fathers, as the Jews can. Therefore we were excluded from the promises of God, for he only made them to the children of the fathers. To show this is the correct interpretation, let us study Deuteronomy 10: 13-22.

I won't quote the whole passage but I strongly urge you to look it up. Here is a portion of the text:

> Only the Lord had a delight in thy fathers to love them, and he chose their seed after them, even you above all people, as it is this day. Circumcise therefore the foreskin of your heart, and be no more stiffnecked. For the Lord your God is God of gods, and Lord of lords, a great God, a mighty, and a terrible, which regardeth not persons, nor taketh reward: he doth execute the judgment of the fatherless and widow, and loveth the stranger, in giving him food and raiment.
>
> Love ye therefore the stranger: for ye were strangers in the land of Egypt (Deut. 10:15-19).

Notice that Moses mentions how the Jews should love the strangers because the Lord did and provided them with food and raiment. If the Lord was not a respecter of persons, then the Jews should not be either, even though they indeed had been chosen by God above all other people to inherit the promises given to their fathers. Those Gentiles, that is, the strangers, were fatherless and poor; they should be taken in under the wing of the Jews and helped.

It is for this reason that Paul said to the Gentile believers,

> And if ye be Christ's, then are ye Abraham's seed, and heirs according to the promise (Gal. 3:29—see also the entire epistle).

C. Leo Jordan

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